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# Comparison of Two Kernels for the Modified Wigner Distribution Function

**A Presentation to the Society of Photo-Optical Instrumentation Engineers International Symposium on Optical Applied Science & Engineering, Conference 1566 on Advanced Signal Processing Algorithms, Architectures, and Implementations, July 21-26 1991, San Diego, California**

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**Newport, Rhode Island • New London, Connecticut**

## Preface

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This document contains the lecture presentation of the paper entitled "Comparison of Two Kernels for the Modified Wigner Distribution Function," given at the Society of Photo-Optical Instrumentation Engineers International Symposium on Optical Applied Science and Engineering, Conference 1566 on Advanced Signal Processing Algorithms, Architectures, and Implementations, July 21-26, 1991, San Diego, California.

We compare the modified Wigner distribution functions obtained via the Choi-Williams kernel and its rotation, as well as by the tilted Gaussian kernel. Based on several commonly used examples, we demonstrate that the modified Wigner distribution obtained via the Gaussian kernel can minimize the artifacts more effectively and has the capability of selectively filtering out undesired components.

**14. SUBJECT TERMS**

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Gaussian Kernel
Choi-Williams Kernel
Interference
Time-Frequency Representation
Ambiguity Function

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# COMPARISON OF TWO KERNELS FOR THE MODIFIED WIGNER DISTRIBUTION FUNCTION

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# OBJECTIVE

- COMPARE TILTED GAUSSIAN AND CHOI-WILLIAMS KERNELS IN TERMS OF MODIFIED WIGNER DISTRIBUTIONS FOR MULTI-COMPONENT SIGNALS.

CRITERIA FOR THE COMPARISON ARE

- CROSS-TERM SUPPRESSION
- DESIRED COMPONENT EXTRACTION

# OUTLINE

- WIGNER DISTRIBUTION & COMPLEX AMBIGUITY FUNCTION
- MODIFIED WIGNER DISTRIBUTION

  TILTED GAUSSIAN KERNEL

  CHOI-WILLIAMS KERNEL
- ROTATION OF KERNEL
- EXAMPLES
- CONCLUSION

# WIGNER DISTRIBUTION & COMPLEX AMBIGUITY FUNCTION

**TCF:** $$R(t,\tau) = s\left(t+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right)$$

**SCF:** $$\Phi(\nu,f) = S\left(f+\frac{\nu}{2}\right) S^*\left(f-\frac{\nu}{2}\right)$$

**WDF:** $$W(t,f) = \int R(t,\tau) \exp(-j2\pi f\tau)\, d\tau$$
$$= \int \Phi(\nu,f) \exp(j2\pi\nu t)\, d\nu$$

**CAF:** $$\chi(\nu,\tau) = \int R(t,\tau) \exp(-j2\pi\nu t)\, dt$$
$$= \int \Phi(\nu,f) \exp(j2\pi f\tau)\, df$$

$$W(t,f) = \iint \chi(\nu,\tau) \exp(j2\pi t\nu) \exp(-j2\pi f\tau)\, d\tau d\nu$$

# WIGNER DISTRIBUTION & COMPLEX AMBIGUITY FUNCTION

| WDF | | CAF |
|---|---|---|
| AUTO-TERMS | ⟷ | CONCENTRATED NEAR ORIGIN |
| CROSS-TERMS | ⟷ | SHIFTED AWAY FROM ORIGIN |

THE AUTO-TERMS AND CROSS-TERMS IN WDF, (t,f) DOMAIN, ARE ESSENTIALLY SEPARATED IN CAF, $(\nu,\tau)$ DOMAIN. THE SEPARATION OF THE AUTO-TERMS AND CROSS-TERMS IN CAF DOMAIN MAKES $(\nu,\tau)$ DOMAIN IDEAL FOR KERNEL DESIGN AND FILTERING.

# MODIFIED TIME-FREQUENCY REPRESENTATIONS

**modified CAF:**

$$\hat{\chi}(\nu,\tau) = \chi(\nu,\tau)\,\tilde{v}(\nu,\tau)$$

**modified SCF:**

$$\hat{\Phi}(\nu,f) = \Phi(\nu,f) \overset{f}{\oplus} \tilde{V}(\nu,f)$$

**modified TCF:**

$$\hat{R}(t,\tau) = R(t,\tau) \overset{t}{\oplus} v(t,\tau)$$

**modified WDF:**

$$\hat{W}(t,f) = W(t,f) \overset{tf}{\oplus} V(t,f)$$

**KERNEL IN EACH DOMAIN**

**SCF:** $\tilde{V}(\nu,f) = \int \tilde{v}(\nu,\tau) \exp(-j2\pi f\tau)\, d\tau$

**TCF:** $v(t,\tau) = \int \tilde{v}(\nu,\tau) \exp(j2\pi\nu t)\, d\nu$

**WDF:** $V(t,f) = \int v(t,\tau) \exp(-j2\pi f\tau)\, d\tau$

$$= \int \tilde{V}(\nu,f) \exp(j2\pi\nu t)\, d\nu$$

$$= \int\!\!\int \tilde{v}(\nu,\tau) \exp(j2\pi\nu t - j2\pi f\tau)\, d\nu\, d\tau$$

$\overset{x}{\oplus}$ **DENOTES CONVOLUTION ON X**

# TILTED GAUSSIAN KERNEL

**CAF:** $$\tilde{v}(\nu,\tau) = \exp\left[-\pi\left(\frac{\nu^2}{B^2}+\frac{\tau^2}{D^2}+2r\frac{\nu}{B}\frac{\tau}{D}\right)\right]$$

**WDF:** $$V(t,f) = \frac{BD}{\sqrt{1-r^2}}\exp\left[-\frac{\pi}{1-r^2}\left(B^2t^2+D^2f^2+2rBtDf\right)\right]$$

**ARBITRARY B,D , AND r**

**DIMENSIONLESS TILT PARAMETER $|r|<1$**

# CHOI-WILLIAMS KERNEL

**CAF:** $\tilde{v}(\nu,\tau) = \exp(-\nu^2\tau^2/\sigma^2), \quad \sigma>0$

**WDF:**

$$V(t,f) = 2\pi^{1/2}\sigma\int_{0^+}^{\infty}\cos(2\pi\nu t)\exp(-\pi^2\sigma^2 f^2/\nu^2)\,\frac{d\nu}{\nu}$$

$$= 2\pi^{1/2}\sigma\int_{0^+}^{\infty}\cos(2\pi f\tau)\exp(-\pi^2\sigma^2 t^2/\tau^2)\,\frac{d\tau}{\tau},$$

$t\neq 0,\ f\neq 0$

# ROTATION OF KERNEL

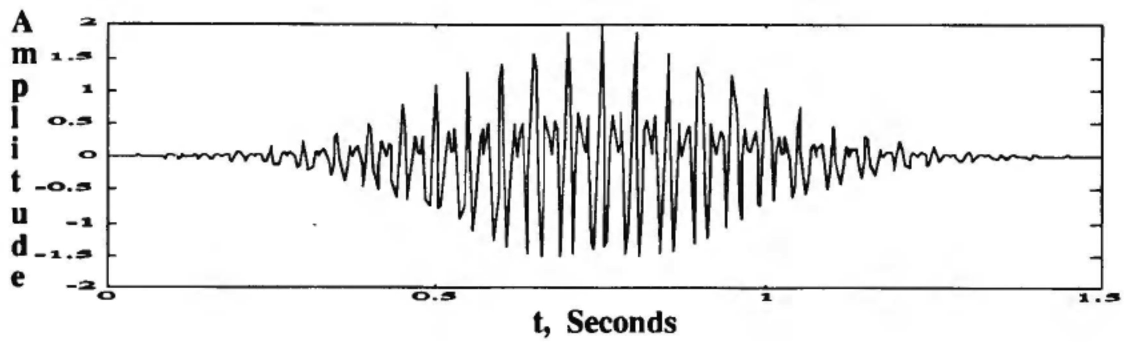
**EXPRESS $\tilde{v}$ IN TERMS OF A NORMALIZED FUNCTION $\tilde{u}$:**

$$\tilde{v}(\nu,\tau) \quad = \quad \tilde{u}(\nu/B,\tau/D)$$

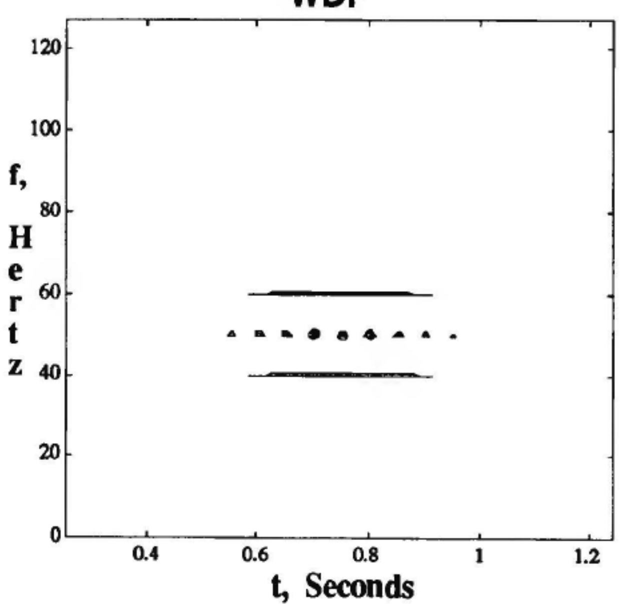
**LET $\theta$ BE THE ANGLE OF ROTATION IN THE $\nu/B$, $\tau/D$ PLANE, C=COS($\theta$), S=sin($\theta$). THEN THE ROTATED WEIGHTING IS**

$$\begin{aligned}\tilde{r}(\nu,\tau) &= \tilde{u}\left(C\frac{\nu}{B}+S\frac{\tau}{D}, C\frac{\tau}{D}-S\frac{\nu}{B}\right)\\ &= \tilde{v}\left(C\nu+S\frac{B}{D}\tau, C\tau-S\frac{D}{B}\nu\right)\end{aligned}$$

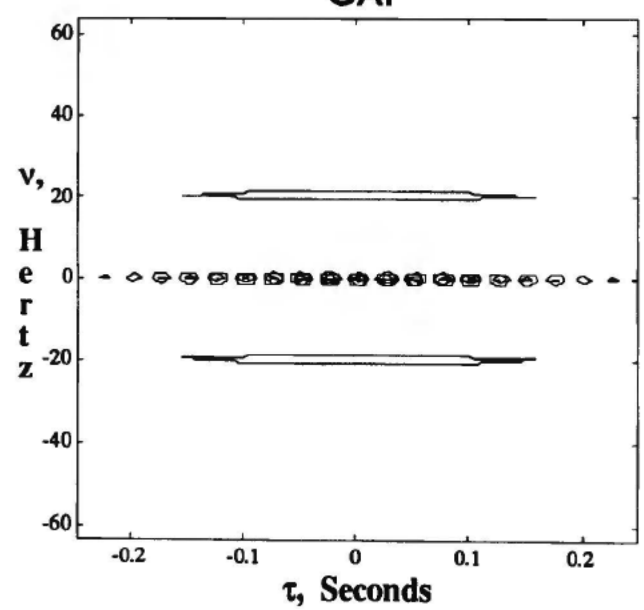
# EXAMPLE 1: TWO AM SINEWAVES




WDF




CAF



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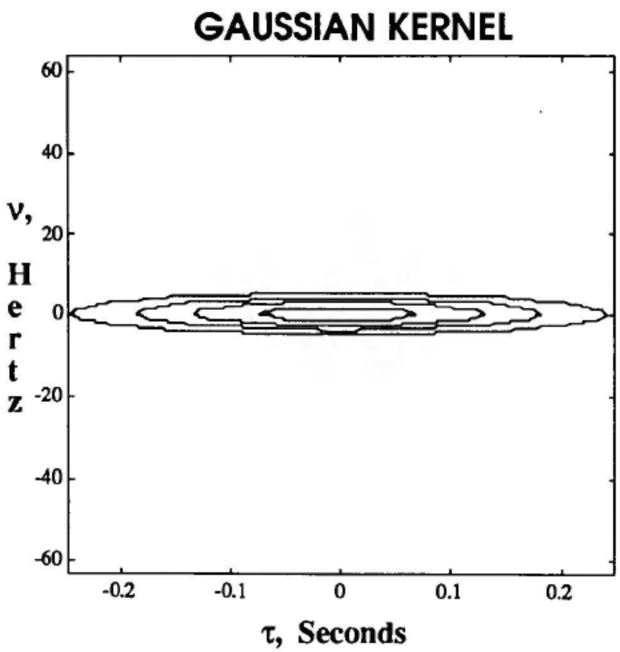


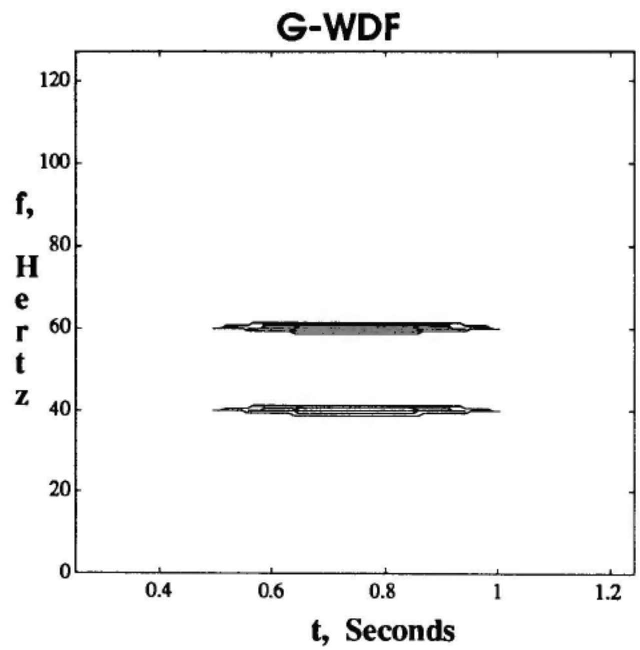

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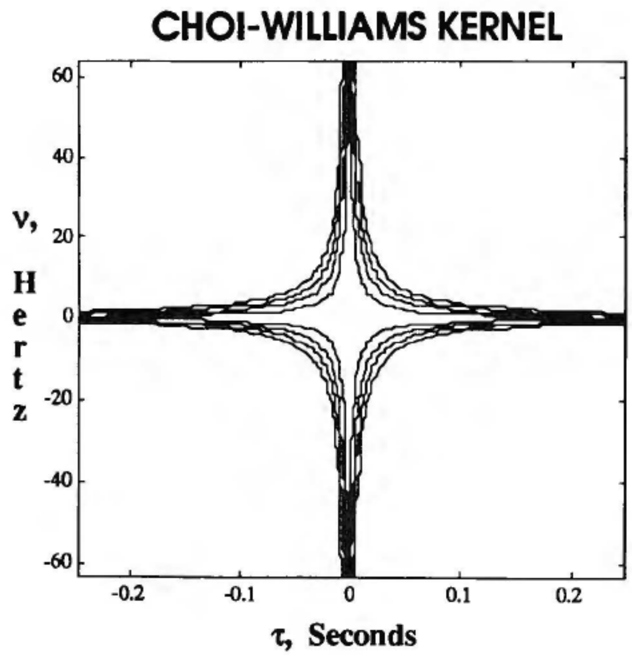
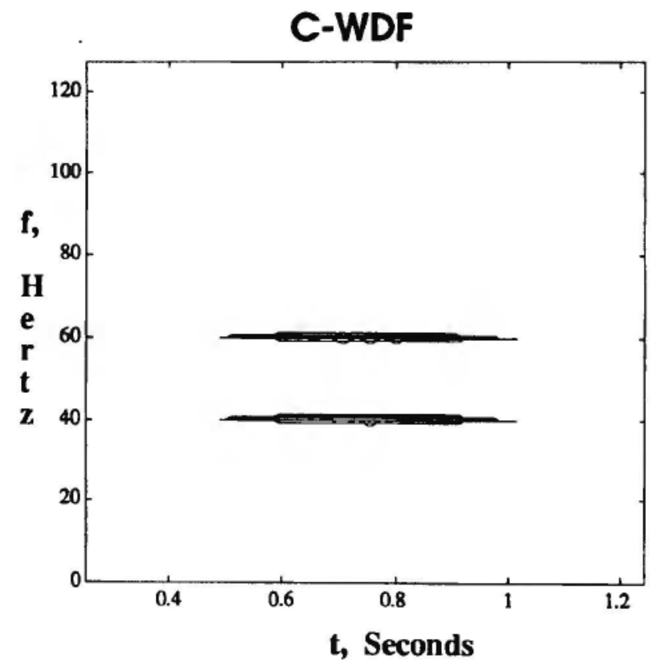
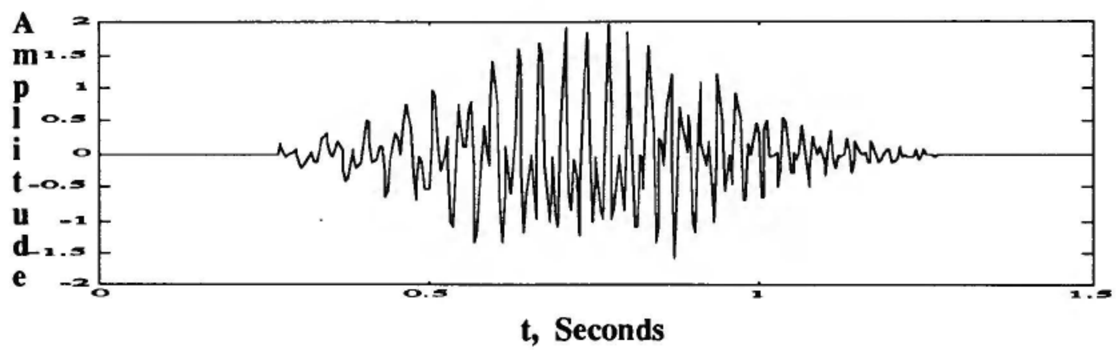


# EXAMPLE 2: TWO PARALLEL LFMS

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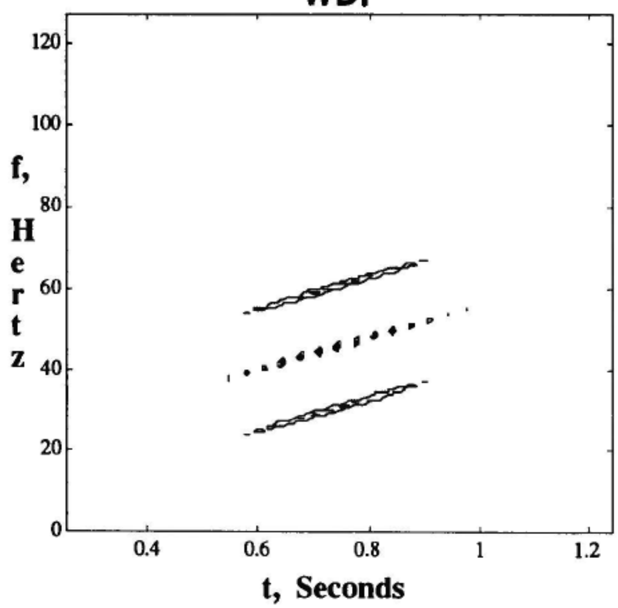


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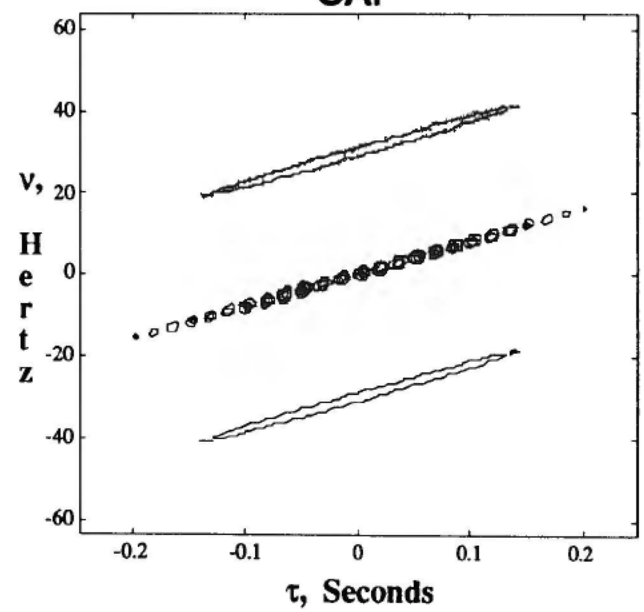

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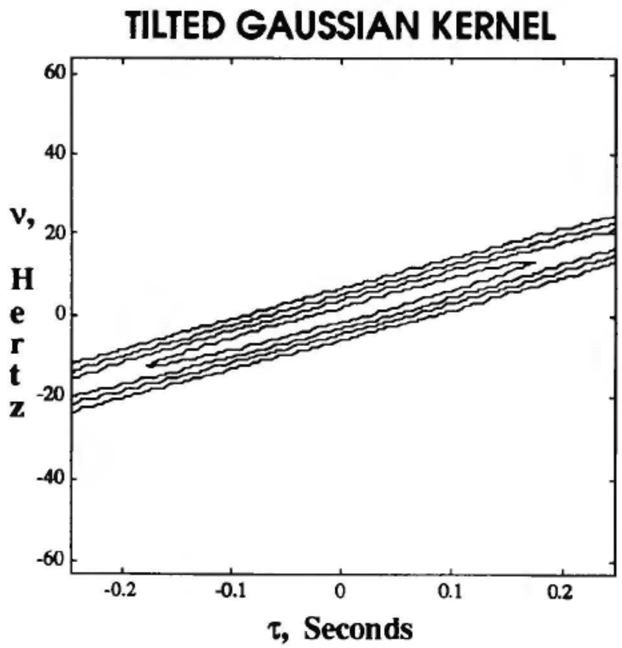
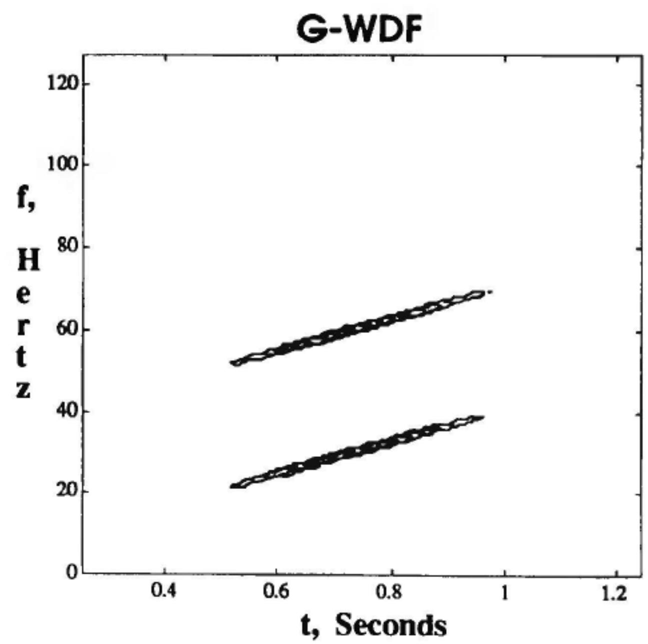
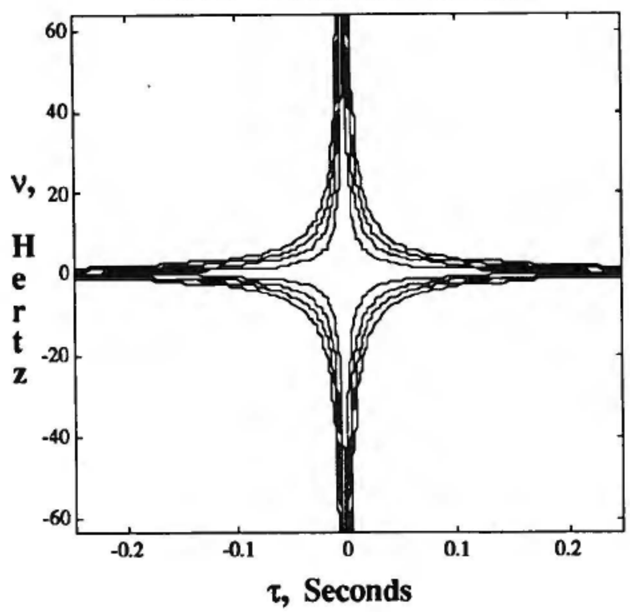


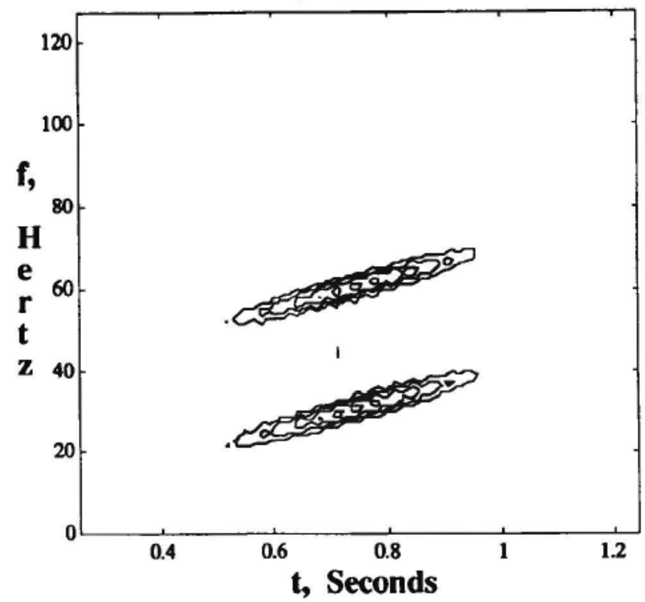



# EXAMPLE 2: TWO PARALLEL LFMS

CHOI-WILLIAMS KERNEL




C-WDF



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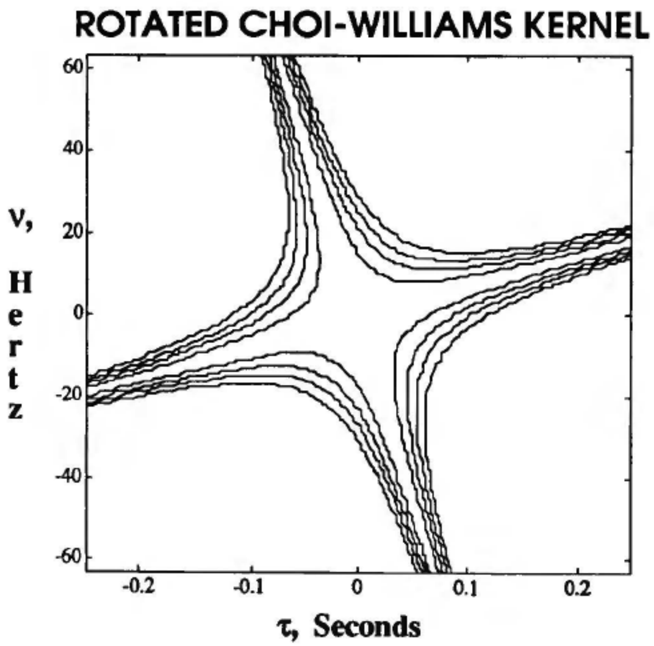
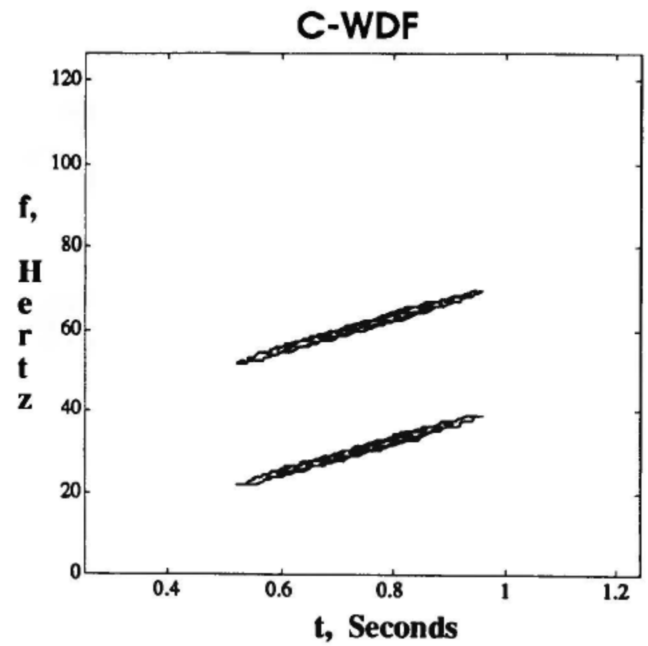
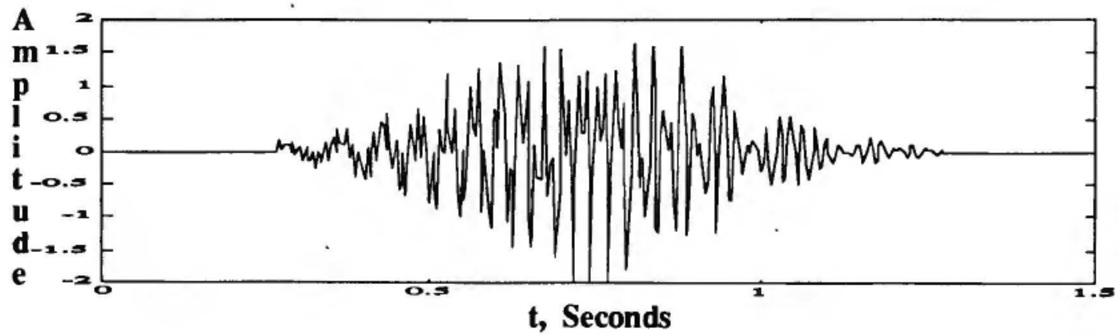
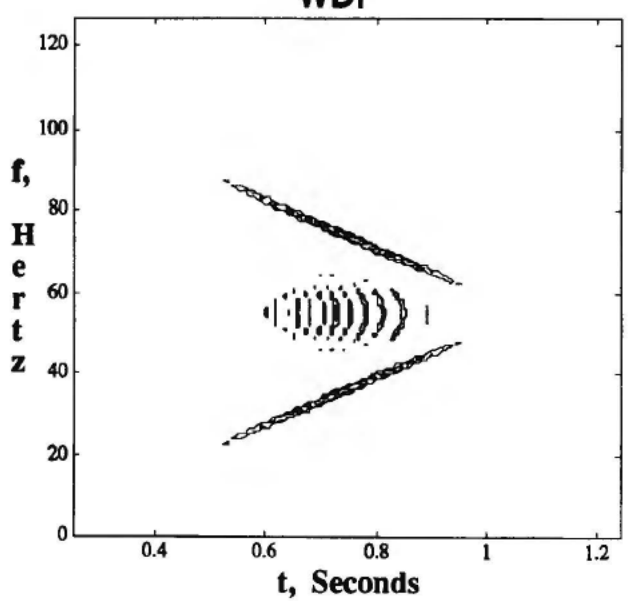


# EXAMPLE 3: UPSLIDE, DOWNSLIDE LFMS




WDF

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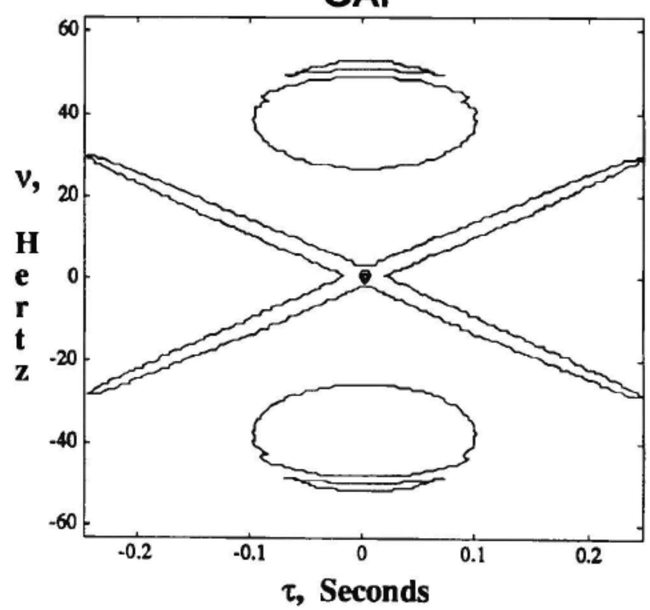
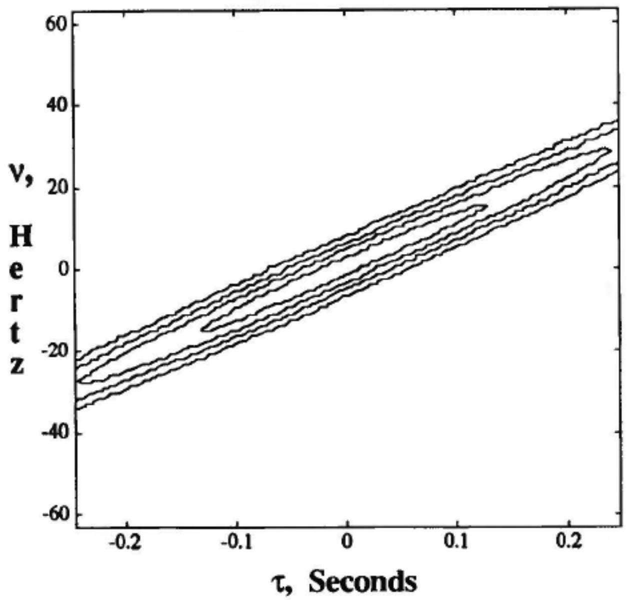
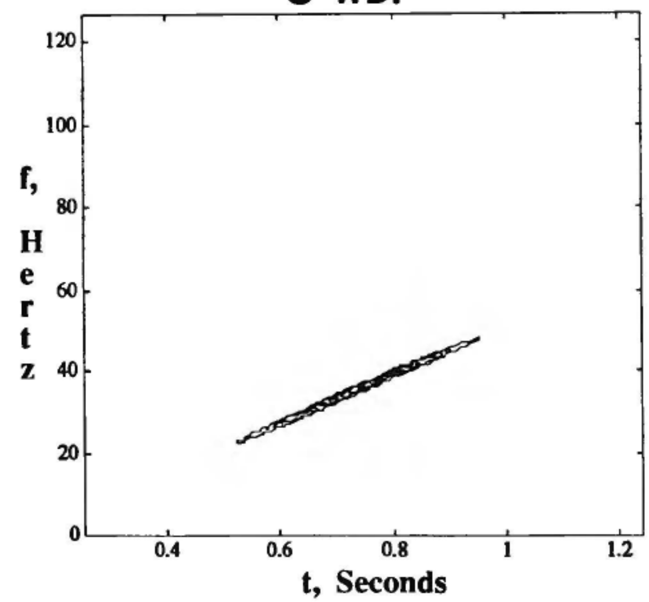


# EXAMPLE 3: UPSLIDE, DOWNSLIDE LFMS

TILTED GAUSSIAN KERNEL

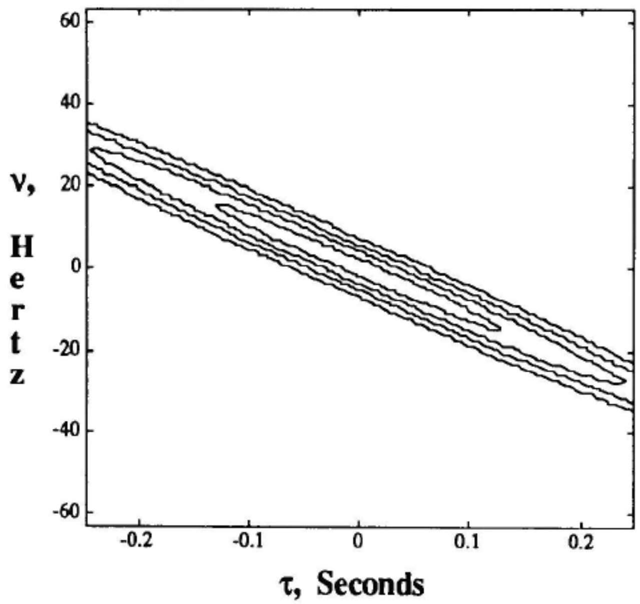



G-WDF



# EXAMPLE 3: UPSLIDE, DOWNSLIDE LFMS

TILTED GAUSSIAN KERNEL

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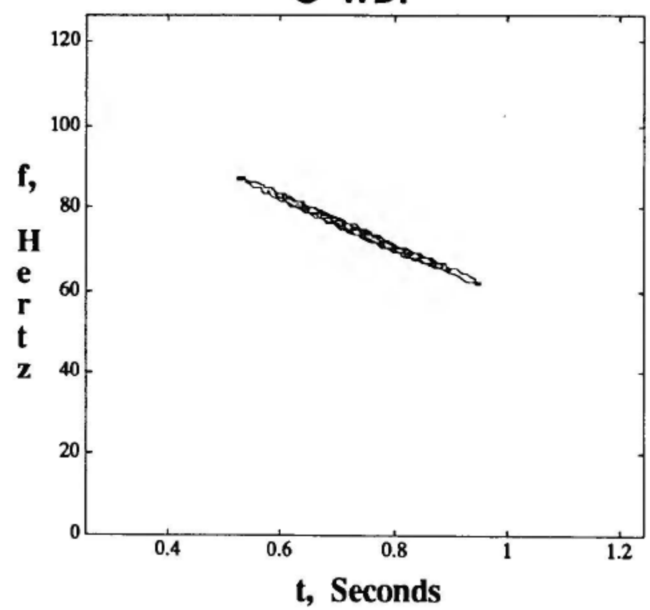
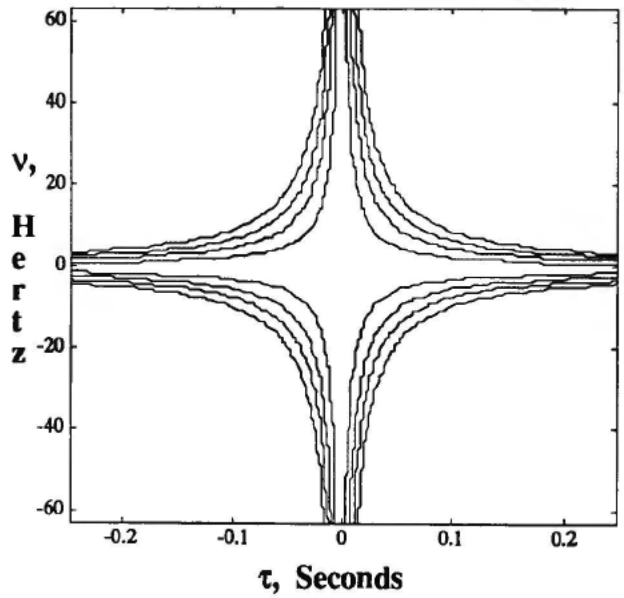


# EXAMPLE 3: UPSLIDE, DOWNSLIDE LFMS

CHOI-WILLIAMS KERNEL

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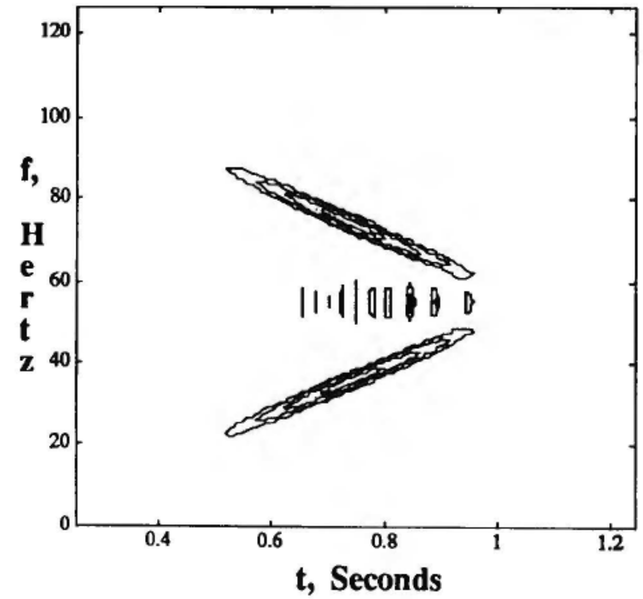
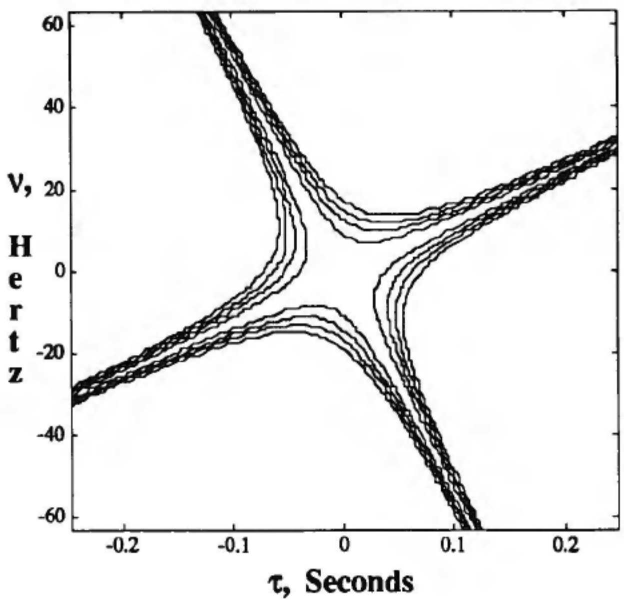


# EXAMPLE 3: UPSLIDE, DOWNSLIDE LFMS

ROTATED CHOI-WILLIAMS KERNEL

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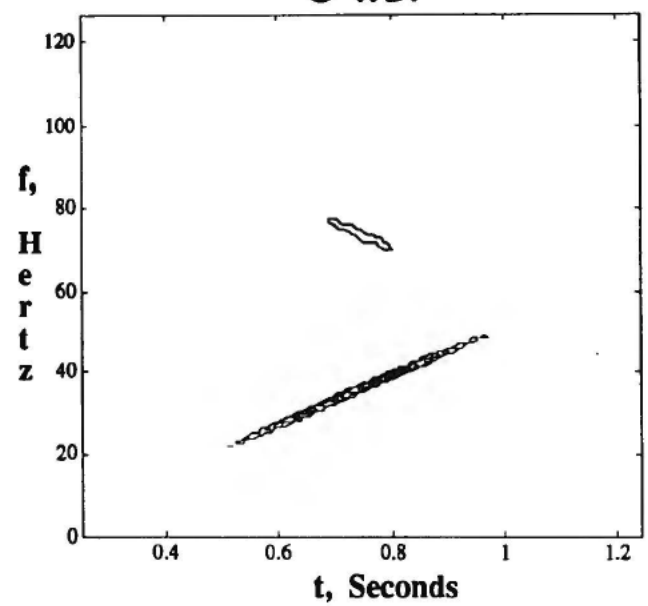

# CONCLUSIONS

## TILTED GAUSSIAN KERNEL

- EFFECTIVELY SUPPRESSES THE CROSS-TERMS AND RECOVERS THE AUTO-TERMS IN ALL CASES CONSIDERED HERE
- EFFECTIVELY EXTRACTS SELECTED COMPONENTS, PROVIDED THE ENERGY DISTRIBUTIONS DO NOT SIGNIFICANTLY OVERLAP IN THE (t,f) DOMAIN

## CHOI-WILLIAMS KERNEL

- EFFECTIVELY SUPPRESSES THE CROSS-TERMS AND RECOVERS THE AUTO-TERMS ONLY FOR THE CASE WHERE THE SLOPES OF THE LINES OF INSTANTANEOUS FREQUENCY ARE ZERO; I.E. NO FM
- HAS NO CAPABILITY TO EXTRACT SELECTED COMPONENTS

# CONCLUSIONS

## ROTATED CHOI-WILLIAMS KERNEL

- SUBSTANTIALLY IMPROVES THE CROSS-TERM SUPPRESSION AND THE AUTO-TERM RECOVERY FOR THE CASE WHERE THE SLOPES OF THE LINES OF INSTANTANEOUS FREQUENCY ARE NON-ZERO

- EXTRACTION OF SELECTED COMPONENTS WITH DISTINCT SLOPES OF THE LINES OF INSTANTANEOUS FREQUENCY CAN BE DONE WITH LIMITED SUCCESS

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